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DIFFRACTION MODEL OF HIGH-ENERGY LEPTONIC INTERACTIONS

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Abstract: We estimate the cross sections for leptonic production of π, ρ and A_1 mesons and total leptonic cross sections using the meson-dominance assumptions. The comparison of our predictions with data on deep electroproduction total cross sections provides a partial check of the ideas of the model. Simple experimental implications of the model are given.

In addition, independently of the model, we show that the experimental test of the PCAC hypothesis in neutrino scattering at small leptonic momentum transfer does not require a cut-off on the scattering angle, but only in the momentum transfer.

1. INTRODUCTION

In this work we estimate the cross sections for leptonic production of π, ρ and A_1 mesons and the total leptonic cross sections in terms of hadronic quantities using the meson-dominance assumption. (A shortened version of this work has already been published [1].)

The success of the 'ρ-photon analogy' or 'vector dominance' [2] in describing the electromagnetic interactions of hadrons, particularly for high-energy photoproduction of vector mesons, suggests that a similar model be used for high-energy electroproduction or neutrino production when the electron or neutrino produces a hadronic system of high mass. The recent measurements of 'deep' electroproduction [3] total cross sections should, by the diffraction analogy, predict something about electro-ρ-production. As for the weak vector current, it is by CVC just the isotopic rotation of the electromagnetic current. If the action of the electromagnetic (iso-vector) current at high energy is described by the scattering of a ρ^0 meson, then the weak vector current is almost necessarily described by the scattering of a $\rho^{\pm}$ meson.

To generalize this idea to the axial weak current, we can use the A_1 meson as a chiral partner of the ρ; but since the current is not conserved,

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there must be a component in the current beyond that corresponding to a simple spin-one particle. The simplest assumption is that there is also a component like the gradient of the pion field. Hence we assume the axial current is proportional to a sum of the gradient of the π-field and the A_1 field. For the axial current, the PCAC hypothesis plays a role like that of current conservation for the vector current and gives certain restrictions on the relation between pion and A_1 contributions. It turns out that these restrictions play an important part near a momentum transfer $q^2 \simeq 0$.

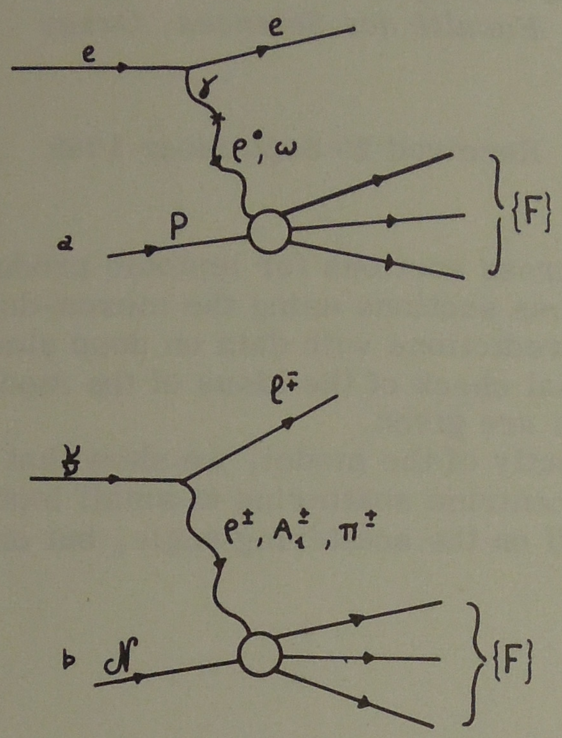

Fig. 1. Meson dominance in electroproduction (a) and neutrino production (b).

Our model can be represented by fig. 1 or rather by the sum of two such graphs in electroproduction, the q line representing ρ^0 and ω. In neutrino production there are three contributions coming from the $\rho^\pm$, $A_1^\pm$, $\pi^\pm$ exchange that correspond to ν or $\bar{\nu}$ scattering. We then use experimental information or plausible guesses for the high-energy reactions $(\rho, A_1, \pi) + N \rightarrow \{F\}$. In particular the electroproduction data give us essentially the ρ-contributions summed over all $\{F\}$. (From photoproduction the ω-contribution is $\frac{1}{9}$ that of the ρ and Φ is negligible.)

Sect. 2 is devoted to a discussion of the kinematics and the derivation of the general formulae of lepton scattering on a nucleon or nucleus when only the leptonic momenta are measured.

In subsect. 2.1 we study the vector current contribution. In (i) we show that from CVC the scattering can be expressed in terms of two real structure functions that depend on q^2 and ν, the leptonic momentum squared and energy transfer, and derive the cross sections for electroproduction and weak vector production of an hadronic system $\{F\}$. In (ii) we introduce explicitly ρ-dominance for the vector current, express the scattering in terms of polarized transverse and longitudinal cross sections, and rewrite in the framework of this model the electromagnetic and weak vector cross section for production of $\{F\}$.

In subsect. 2.2, we study in the same way the axial current contribution.

In (i) we first show that the scattering can be expressed in terms of four real structure functions depending on q^2 and ν. We discuss the constraints coming from PCAC and the lepton mass-terms contributions. We obtain in particular a generalized Adler formula [4] for weak scattering at small q^2 and compare it to recent data on forward weak total cross sections; finally, we give the axial contribution for the weak production of $\{F\}$ at large q^2. In (ii) we then introduce explicitly meson dominance for the axial current. We translate the general results in the framework of the model: that is in terms of the transverse and longitudinal A_1 and π-scattering. Finally, in subsect. 2.3 we study the interference between the vector and axial currents.

Sect. 3 is devoted to the applications of the model, we give the assumptions required and estimate the various quantities introduced in sect. 2.

Sect. 4 is devoted to the results. In subsect. 4.1 the comparison of our predictions with recent data on deep-electroproduction total cross section [3] provides a partial check of the ideas of the model. We compare our results with others derived from different hypotheses. We also estimate ρ^0 electroproduction, and the charged π electroproduction.

Subsect. 4.2 is devoted to neutrino reactions, starting with weak ρ-production. In particular, we express weak charged ρ-production in terms of ρ^0 electroproduction.

We than examine A_1 weak production, π-weak production, total weak meson production, and the total weak cross section. We also discuss the asymptotic behaviour in energy predicted for the total cross section.

2. GENERAL FORMULAE FOR LEPTON SCATTERING ON A NUCLEON OR NUCLEUS

2.1. *Vector current*

(i) Kinematics and general formulae. Let us consider for definiteness the production of a given hadronic state $\{F\}$ of total momentum P' and (invariant mass)$^2 = \omega^2$ by a vector current of momentum q. The notations are summarized in fig. 2.

Here p and p' are the momenta of the initial and final leptons, $\nu = E - E'$ is the transfer of leptonic energy in the lab system. For simplicity polarization indices and momenta of individual particles of $\{F\}$ are omitted since they will not be measured. The scattering matrix element is given by

$$\ell_\mu V_\mu^{FI} ,$$

where V_μ^{FI} is the hadronic current matrix element and ℓ_μ the leptonic, and

$$\frac{e}{q^2} \bar{u}(p')\gamma_\mu u(p) ,$$

$$\frac{G}{\sqrt{2}} \bar{u}(p')\gamma_\mu(1+\gamma_5)u(p) ,$$

for electromagnetic and weak scattering.

The average over initial states and sum over final ones of the transition matrix element squared can be written, up to a kinematical factor, as $\ell_{\mu\nu}\,\mathcal{M}_{\mu\nu}$ where $\ell_{\mu\nu}$ is a purely leptonic tensor and

$$\ell_{\mu\nu} = \frac{e^2}{2q^4}\left(p_\mu p'_\nu + p_\nu p'_\mu - (p\cdot p' + m^2)\,\delta_{\mu\nu}\right) \text{ for electromagnetic scattering,}$$

$$\ell_{\mu\nu} = G^2\left(p_\mu p'_\nu + p_\nu p'_\mu - (p\cdot p')\,\delta_{\mu\nu} \pm \epsilon_{\mu\nu\alpha\beta}\,p_\alpha p'_\beta\right) \text{ for weak scattering ,}$$

the ± sign referring to ν or $\bar{\nu}$ and

$$\mathcal{M}_{\mu\nu} = \overline{\sum_{\mathrm{I}}}\,\sum_{\mathrm{F}} V_\mu^{*\mathrm{FI}}\, V_\nu^{\mathrm{FI}}$$

contains all the information on the hadronic system.

Since Σ_{F} is over all spins and all momenta internal to $\{\mathrm{F}\}$ and $\overline{\Sigma}_{\mathrm{I}}$ is over the spin of the nucleon (or the nucleus) target, $\mathcal{M}_{\mu\nu}$ can only be made from P_μ for the target and q_μ for the momentum carried by the current, four-momentum conservation removing the total four vector of $\{\mathrm{F}\}$. Using Lorentz invariance, hermiticity, the constraint of current conservation (that states $q_\mu\,\mathcal{M}_{\mu\nu} = q_\nu\,\mathcal{M}_{\mu\nu} = 0$) and the fact that $\mathcal{M}_{\mu\nu}$ must be free from singularities at $q^2 = 0$, one derives the general form of $\mathcal{M}_{\mu\nu}$ [5]):

$$\mathcal{M}_{\mu\nu} = F_0^{\mathrm{V}}(q^2,\nu)\left[\delta_{\mu\nu} - \frac{P_\mu q_\nu + P_\nu q_\mu}{P\cdot q} + q^2\frac{P_\mu P_\nu}{(P\cdot q)^2}\right]$$

$$+ F_1^{\mathrm{V}}(q^2,\nu)\left[q_\mu q_\nu - q^2\,\frac{(P_\mu q_\nu + P_\nu q_\mu)}{P\cdot q} + q^4\,\frac{P_\mu P_\nu}{(P\cdot q)^2}\right], \qquad (1)$$

where the F_i^{V} are real positive quantities depending on q^2 and ν.

Electromagnetic scattering. For electromagnetic scattering the terms proportional to q_μ or q_ν do not contribute because $q_\mu \cdot \ell_\mu = 0$ and we get:

$$\frac{\mathrm{d}^2\sigma(\mathrm{e}\to\mathrm{F})}{\mathrm{d}q^2\,\mathrm{d}\nu} = \frac{2\alpha^2}{q^4}\,\frac{1}{E^2}\left[F_0^{\mathrm{V}}\,q^2 + (F_0^{\mathrm{V}} + q^2 F_1^{\mathrm{V}})\,\frac{q^2}{2\nu^2}\,(4EE' - q^2)\right], \qquad (2)$$

or in terms of the usual structure functions W_1 and W_2 in the lab system [6]:

$$\frac{\mathrm{d}^2\sigma(\mathrm{e}\to\mathrm{F})}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{\alpha^2}{4E^2\sin^4\frac{1}{2}\theta}\left[W_2\cos^2\tfrac{1}{2}\theta + 2W_1\sin^2\tfrac{1}{2}\theta\right], \qquad (3)$$

where θ is the lepton scattering angle in the lab system and E and E' the incident and final lepton energies.

Weak vector scattering. For weak vector scattering the terms proportional to q_μ (or q_ν) give rise to contributions proportional to the lepton mass m that do not affect the cross section greatly.

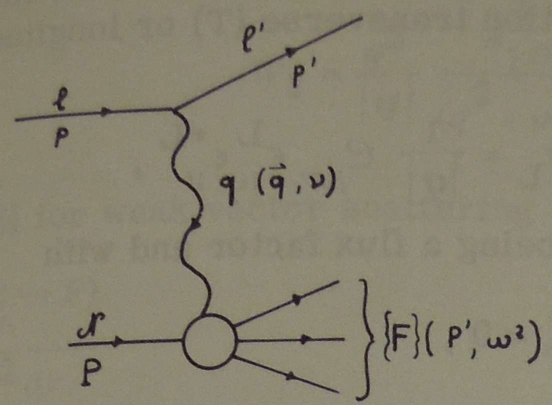

Fig. 2. Kinematics.

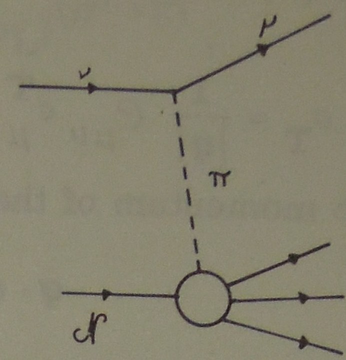

Fig. 3. One-pion-exchange contribution.

We shall see later that the order of magnitude of the neglected lepton mass terms is something like m^2/m_ρ^2 where m_ρ is the mass of the ρ-meson, that is less than a few percent.

Thus we have, with no average on the ν-spin:

$$\frac{d^2\sigma(\nu\to F)}{dq^2\, d\nu} = \frac{G^2}{4\pi^2 E^2}\left[F_0^V q^2 + (F_0^V q^2 F_1^V)\frac{q^2}{2\nu^2}(4EE' - q^2)\right], \tag{4}$$

for the vector current only.

(ii) Meson dominance for the vector current. Let us now introduce explicitly the hypothesis of the ρ-dominance for the vector current. The hadronic isovector, vector current takes the form, with $g_{\gamma\rho}$, $g_{\ell\rho}$ the effective $\gamma\rho$ and lepton-ρ couplings:

$$V_\mu = \frac{g_{\gamma\rho} \text{ or } g_{\ell\rho}}{q^2 + m_\rho^2}\left[\delta_{\mu\lambda} + \frac{q_\mu q_\lambda}{m_\rho^2}\right] T_\lambda^{(\rho\to F)} \text{(electromagnetic}^\dagger \text{ or weak current)} .$$

Where $\epsilon_\lambda^i T_\lambda^{(\rho\to F)}$ describes the production of a given hadronic state $\{F\}$ by a ρ-meson of mass q^2, of momentum q and polarization i. The ρ is taken to be universally coupled to the conserved isospin current. The vector meson then must also obey $q_\lambda T_\lambda^{(\rho\to F)} = 0$ so that $\mathcal{M}_{\mu\nu}$ is simply given by:

$$\mathcal{M}_{\mu\nu} = \frac{g_{\gamma\rho^0}^2 (\text{or } g_{\ell\rho}^2)}{(q^2 + m_\rho^2)^2} \underbrace{\overline{\sum_I}\sum_F T_\mu^{(\rho\to F)} T_\nu^{*(\rho\to F)}}_{\mathcal{C}_{\mu\nu}} .$$

† For the electromagnetic current the ω and φ are to be included in the theory and they play, for the isoscalar part of the electromagnetic current, the same role as the ρ^0 does for the isovector. However, from photoproduction results the ω-contribution is $\frac{1}{9}$ that of the ρ^0 and that of the Φ is negligible [2]. So, for simplicity, we will consider the ρ^0 only, although the ω has been explicitly taken into account in the calculations of electroproduction cross sections.

Let us now introduce formally the polarized cross sections to the final set of states $\{F\}$ for incident ρ of polarization transverse (T) or longitudinal (L)

$$\sigma_{\rm T} = \frac{1}{|\boldsymbol{q}|}\, \mathcal{C}_{\mu\nu}\, \epsilon^{\rm T}_{\mu} \epsilon^{*{\rm T}}_{\nu} , \qquad \sigma_{\rm L} = \frac{1}{|\boldsymbol{q}|}\, \mathcal{C}_{\mu\nu}\, \epsilon^{\rm L}_{\mu} \epsilon^{*{\rm L}}_{\nu} , \tag{7}$$

$|\boldsymbol{q}|$, the lab momentum of the virtual ρ, being a flux factor and with

$$\boldsymbol{q} \cdot \varepsilon_{\rm T} , \qquad \epsilon^{\rm o}_{\rm T} = 0 ,$$

$$\epsilon_{\rm L} = \frac{1}{\sqrt{|q^2|}} (q_0 \hat{\boldsymbol{q}}, |\boldsymbol{q}|) , \qquad \epsilon^2_{\rm T,L} = \pm 1 .$$

One easily verifies that:

$$\text{(a)} \quad F^{\rm V}_{\rm o}(q^2, \nu) = |\boldsymbol{q}|\, \sigma_{\rm T} \frac{g^2_{\gamma\rho^{\rm o}} \text{ or } g^2_{\ell\rho}}{(q^2 + m^2_\rho)^2} ,$$

$$\text{(b)} \quad \frac{|\boldsymbol{q}|^2}{\nu^2} [F^{\rm V}_{\rm o} + F^{\rm V}_1 q^2] = |\boldsymbol{q}|\, (\sigma_{\rm T} + \sigma_{\rm L}) \frac{g^2_{\gamma\rho^{\rm o}} (\text{or } g^2_{\ell\rho})}{(q^2 + m^2_\rho)^2} ,$$

$$\text{(c)} \quad \frac{g^2_{\gamma\rho^{\rm o}} (\text{or } g^2_{\ell\rho})}{(q^2 + m^2_\rho)^2}\, |\boldsymbol{q}|\, \sigma_{\rm L} = \frac{q^2}{\nu^2} (F^{\rm V}_{\rm o} + |\boldsymbol{q}|^2 F^{\rm V}_1) . \tag{8}$$

Let us remark that $\mathcal{M}_{\mu\nu}$ is not singular, one verifies (looking at (8c)) that $\sigma_{\rm L}(q^2) \sim C q^2$ as q^2 tends to zero, as is expected for zero-mass vector particle, and thus that the first term in q^2 near $q^2 = 0$ is given by $\sigma_{\rm T}(q^2 = 0)$, as expected from the general theorems on electroproduction.

The differential cross sections now take the form

(i) for electromagnetic scattering (see (2) and (8)):

$$\frac{{\rm d}^2\sigma^{e \to F}}{{\rm d}q^2\, {\rm d}\nu}$$

$$= \frac{\alpha^2}{q^4} \frac{2}{E^2} g^2_{\gamma\rho^{\rm o}} \frac{q^2}{(q^2 + m^2_\rho)^2} |\boldsymbol{q}| \left[\sigma_{\rm T}^{(\rho^{\rm o} \to {\rm F})} + \frac{(\sigma_{\rm T} + \sigma_{\rm L})^{(\rho^{\rm o} \to {\rm F})}}{2|\boldsymbol{q}|^2} (4EE' - q^2)\right] , \tag{9}$$

or from (3) and (8)

$$W_1 = |\boldsymbol{q}| \frac{g^2_{\gamma\rho^{\rm o}}}{(q^2 + m^2_\rho)^2} \frac{1}{\pi} \sigma_{\rm T}^{(\rho^{\rm o} \to {\rm F})} , \tag{10a}$$

$$W_2 = \frac{q^2}{|\boldsymbol{q}|} \frac{g^2_{\gamma\rho^0}}{(q^2+m^2_\rho)^2} \frac{1}{\pi} (\sigma_T + \sigma_L)^{(\rho^0 \to F)} , \tag{10b}$$

(ii) for weak vector scattering

$$\frac{d^2\sigma^{\left(\substack{\nu\\ \bar\nu} \to F\right)}}{dq^2\, d\nu} = \frac{G^2}{\pi^2} \frac{1}{4E^2} g^2_{\ell\rho} \frac{q^2}{(q^2+m^2_\rho)^2} |\boldsymbol{q}| \left[\sigma_T^{(\rho^\pm \to F)} + \frac{(\sigma_T+\sigma_L)^{(\rho^\pm \to F)}}{2|\boldsymbol{q}|^2} (4EE' - q^2)\right] . \tag{11}$$

Let us remark that within the model the order of magnitude of the lepton mass terms becomes obviously of the order of m^2/m^2_ρ and justifies their neglect.

The connection in the cross section between a weak and electromagnetic process induced by the vector current is then

$$\frac{\alpha^2}{q^4} g^2_{\gamma\rho^0} \sigma^{(\rho^0 \to F)} \to \frac{G^2}{8\pi^2} g^2_{\ell\rho^\pm} \sigma^{(\rho^\pm \to F)} ,$$

where (see subsect. 3.2): $g_{\ell\rho^\pm} = \sqrt{2}\, g_{\gamma\rho^0}$. The assumption that ρ^+ and ρ^0 scattering is the same involves more than CVC of course, and is a feature of this kind of model.

Before seeing how we can get some quantitative estimates from the model and particularly the assumption needed for it, let us derive in the same way the general formulae for the axial current.

2.2. *Axial current*

(i) General formulae. The axial current treatme nt is quite close to that of the vector current and we refer to subsect. 2.1 for definitions and details. However, it will differ from that of the vector on two points, both concerning small q^2. The first one comes from the fact that the axial current is only partially conserved and the second one from the lepton mass terms.

The general form of the hadronic tensor is

$$\mathcal{M}^A_{\mu\nu} = \overline{\sum_I} \sum_F A^{*FI}_\nu A^{FI}_\mu ,$$

then the Σ_F is over all spins and momenta internal to $\{F\}$ and $\overline{\Sigma}_I$ over spins of the target, $\mathcal{M}^A_{\mu\nu}$ takes the form

$$\mathcal{M}^A_{\mu\nu} = F^A_0 \delta_{\mu\nu} + F^A_1 P_\mu P_\nu + F^A_2 (P_\mu q_\nu + P_\nu q_\mu) + F^A_3 q_\mu q_\nu , \tag{13}$$

where $F^A(q^2, \nu)$ are real, and positive, depending on q^2 and ν.

The constraint of PCAC states that

$$\partial_\mu A_\mu = g_{\ell\pi}\Phi \ , \tag{14}$$

where Φ is the π-field and $g_{\ell\pi}$ is the coupling constant for weak π-decay. It implies for $\mathcal{M}^{\mathrm{A}}_{\mu\nu}$ that

$$q_\mu \mathcal{M}^{\mathrm{A}}_{\mu\nu} q_\nu = g^2_{\ell\pi} \frac{1}{|\boldsymbol{q}|} \sigma(\pi \to \mathrm{F}) \ . \tag{15}$$

At $q^2 = 0$ all terms vanish in the left-hand side of (15) except the $P_\mu P_\nu$ term, so F^{A}_1 is fixed

$$F^{\mathrm{A}}_1 (P \cdot q)^2 \underset{q^2 \approx 0}{\to} g^2_{\ell\pi} \frac{1}{|\boldsymbol{q}|} \sigma(\pi \to \mathrm{F}) \ . \tag{16}$$

That is for the differential cross section near $q^2 = 0$

$$\frac{\mathrm{d}^2\sigma^{(\nu \to \mathrm{F})}}{\mathrm{d}q^2\,\mathrm{d}\nu} \to \frac{G^2}{2\pi^2} \frac{1}{\nu} g^2_{\ell\pi} \sigma(\pi \to \mathrm{F}) \frac{E'}{E} + \text{lepton mass terms} \ .$$

This is precisely Adler's relation [4].

The lepton mass terms come from the q_μ terms in (13) not fixed by PCAC. However, for low q^2 these terms proportional to the lepton mass squared m^2 can play a role since the π gives the dominant contribution to the q_μ or q_ν terms in (13).

For clarity let us write explicitly in A_μ the one-π-exchange interaction as described in fig. 3.

Then the matrix element to $\{\mathrm{F}\}$ is

$$A^{\mathrm{F}}_\mu = A^{\mathrm{F}'}_\mu + g_{\ell\pi} \frac{q_\mu}{q^2 + m^2_\pi} T^{(\pi \to \mathrm{F})} \ .$$

We will neglect all terms in m^2 except those appearing over the π-propagator, presumably the only big factor at small q^2. Note the propagator drops rapidly with q^2 so that away from very small q^2, only the $\delta_{\mu\nu}$ and $P_\mu P_\nu$ terms are relevant in (13) for neutrino scattering, as in the vector case (see (4)).

Let us consider the part of $\mathcal{M}^{\mathrm{A}}_{\mu\nu}$ that contains the one-pion-exchange contribution, at least once. It can be written as

$$\frac{g_{\ell\pi}}{(q^2 + m^2_\pi)} \overline{\sum_{\mathrm{I}}} \sum_{\mathrm{F}} [q_\mu T^{(\pi \to \mathrm{F})} A^{*\mathrm{F}}_\nu + q_\nu T^{*(\pi \to \mathrm{F})} A^{\mathrm{F}}_\mu] \ , \tag{17}$$

making use of the same procedure as before $\overline{\Sigma}_{\mathrm{I}} \Sigma_{\mathrm{F}} T^{*(\pi \to \mathrm{F})} A^{\mathrm{F}}_\mu$ can be written as†

$$\overline{\sum_{\mathrm{I}}} \sum_{\mathrm{F}} T^{*(\pi \to \mathrm{F})} A^{\mathrm{F}}_\mu = b^{\mathrm{A}}_0 P_\mu + b^{\mathrm{A}}_1 q_\mu + g_{\ell\pi} \frac{q_\mu}{q^2 + m^2_\pi} \overline{\sum_{\mathrm{I}}} \sum_{\mathrm{F}} |T^{(\pi \to \mathrm{F})}|^2 \ , \tag{18}$$

† We recall that Σ_{F} is over spin and momenta internal to $\{\mathrm{F}\}$.

where the b_i^A are free from singularities at $q^2 = 0$ and the $b_i^A q_\mu$ is the non-pionic q_μ part of A_μ^F.

Let us apply once more the constraint of PCAC (14) in eq. (18), that is

$$\overline{\sum_I} \sum_F T^{*(\pi \to F)} A_\mu^F \cdot q_\mu = g_{\ell\pi} \overline{\sum_I} \sum_F |T^{(\pi \to F)}|^2 . \tag{19}$$

One easily verifies that at $q^2 = 0$ all terms vanish in the left-hand side except the P_μ term, so that one gets b_0^A:

$$b_0^A P \cdot q \underset{q^2=0}{=} g_{\ell\pi} \overline{\sum_I} \sum_F |T^{(\pi \to F)}|^2 . \tag{20}$$

Now the b_1^A term is dropped in comparison with the last term in (18), since it presumably is made of terms containing propagators with masses much greater than m_π. Next taking the contraction of (17) with the leptonic tensor, making use of (18) and (20), one derives the neutrino cross section for small q^2

$$\frac{d^2\sigma^{(\overset{\nu}{\bar\nu} \to F)}}{dq^2 d\nu} = \frac{G^2}{2\pi^2} \frac{1}{\nu} g_{\ell\pi}^2 \sigma^{(\pi \to F)} \left[\frac{E'}{E} - \frac{\nu}{E} \frac{m^2}{q^2 + m_\pi^2} + \frac{\nu^2}{4E^2} \frac{m^2(q^2 + m^2)}{(q^2 + m_\pi^2)^2} \right] , \tag{21}$$

where the second term in [ ] comes from the interference of the pionic contribution with the non-pionic one in (17) (proportional to $b_0^A(P_\mu q_\nu + P_\mu q_\nu)$) and the last one from the 'diagonal' pionic term that gives the main contribution to the $q_\mu q_\nu$ term (see below). We get that for q^2 close to zero the weak production of a system $\{F\}$ is given by the π-production of $\{F\}$ since the vector part gives zero from CVC: this is precisely the Adler's relation [4].

Let us point out that the derivation of (21) makes use only of the PCAC hypothesis (14), which states that $\partial_\mu A_\mu = g_{\ell\pi}\Phi$, and of the dominance of one-pion-exchange for small q^2 in the terms proportional to q_μ or q_ν (lepton mass terms and q^2 terms), so that the domain of validity of (21) is in fact q^2 of the order of few m_π^2. Particularly we never make the assumption that the scattering angle in the lab system was small so that the experimental test of (21) required only a cut-off on q^2 and not on the angle. Moreover, the correct expansion to be used is in fact q^2 rather than θ since the PCAC hypothesis should work only for small q^2 and moreover as we will see later on, the vector contribution and part of the axial (that have been taken to be zero in (21)), rise rapidly with increasing q^2 (see figs. 7 and 8). This seems to be confirmed by the preliminary data on forward ν-scattering in propane [7] where the measured cross section was too large with respect to theory, while when a cut-off on $q^2 (q^2 < 0.1\ (\mathrm{GeV}/c)^2)$ is applied to the selected forward events, agreement is obtained with the corresponding theoretical curves.

Eq. (21) is somewhat more general than the relation already obtained by Adler for forward scattering only, that corresponds to an expansion of (21) with respect to the lepton scattering angle. By use of $q^2 + m^2 \approx$

$\approx EE'\,\theta^2 + m^2\nu/E'$, keeping all terms in $m^2/(q^2+m_\pi^2)$ and in first order in θ^2 one easily derives

$$\frac{d^2\sigma}{dq^2\,d\nu} = \frac{G^2}{2\pi^2} g_{\ell\pi}^2 \frac{E'}{E}\frac{1}{\nu}\sigma^{(\pi\to F)} \left[\left(1-\tfrac{1}{2}m^2\frac{\nu}{E'(q^2+m_\pi^2)}\right)^2 + \frac{\nu^2\theta^2 m^2}{4(q^2+m_\pi^2)^2}\right], \tag{22}$$

which is precisely Adler's relation [4] †. Instead of $g_{\ell\pi}$, Adler, making use of the Goldberger-Treiman relation writes $g_{\ell\pi}$ as

$$g_{\ell\pi} = \sqrt{2}\,\frac{MG_A(0)}{g_r}, \quad \text{with} \quad G_A(0) = 1.18, \quad \text{and} \quad g_r^2/4\pi \simeq 12.5.$$

Note however that our simplified derivation depends on the fact that momenta and polarizations of the hadrons are not measured so that we can construct our quantities explicitly from P and q.

Thus for $q^2 \sim$ few m_π^2 we have the generalized Adler's relation (21) for weak production of an hadronic system $\{F\}$, while in the region $q^2 \gg m_\pi^2$ where the lepton mass terms can be neglected, we have for the axial weak production of $\{F\}$ an expression similar to that for the vector current (4)

$$\frac{d^2\sigma(A)}{dq^2\,d\nu} = \frac{G^2}{4\pi^2E^2}\left[F_0^A q^2 + \tfrac{1}{2}F_1^A M^2(4EE' - q^2)\right]. \tag{23}$$

(ii) Meson dominance for the axial current. We shall now introduce explicitly the meson-dominance hypothesis for the axial current, that is we assume that the axial current is dominated by the A_1 and the π so that it can be written as

$$A_\mu^F = g_{\ell A_1}\frac{\delta_{\lambda\mu} + q_\lambda q_\mu/m_A^2}{q^2+m_A^2}\,T_\lambda^{(A_1\to F)} + g_{\ell\pi}\frac{q_\mu}{q^2+m_\pi^2}\,T^{(\pi\to F)},$$

where as before $\epsilon_\lambda^i\,T_\lambda^{(A_1\to F)}$ (or $T^{(\pi\to F)}$) describes the production of $\{F\}$ by an A_1 of polarization i (or by a π) of mass q^2 and momentum q on a target of momentum $P\cdot m_A$ is the A_1 mass. In the hadronic tensor $\mathcal{M}_{\mu\nu}(A)$ summed over all polarizations and over all internal momenta of $\{F\}$ will appear three types of contributions:

(a) $A_1 \to A_1$ 'diagonal' with respect to the A_1 similar to that for the ρ

$$\frac{g_{\ell A_1}^2}{(q^2+m_A^2)^2}\left[|\boldsymbol{q}|\left[\sigma_T^{(A_1\to F)}\delta_{\mu\nu} + \frac{q^2}{|\boldsymbol{q}|^2}(\sigma_T+\sigma_L)^{(A_1\to F)}\frac{P_\mu P_\nu}{M^2}\right]\right.$$
$$\left. + F_2^A(q_\mu P_\nu + q_\nu P_\mu) + F_3^A q_\mu q_\nu\right], \tag{24}$$

† Up to a factor close to one that is the ratio of flux for a physical π and unphysical one of same energy namely $(\nu^2 - m_\pi^2)^{\frac{1}{2}}/(\nu^2+q^2)^{\frac{1}{2}}$.

where $\sigma_{T,L}^{(A1\to F)}$ are the cross sections for production of $\{F\}$ by an A_1 of polarization transverse (or longitudinal); we are not interested in the q_μ terms since they do not contain the pion propagator.

(b) This is an interference term between the π and the A_1

$$\frac{g_{\ell A_1}}{(q^2+m_A^2)}\{[b_0^A P_\mu + b_1^A q_\mu]\frac{q_\nu}{q^2+m_\pi^2} g_{\ell\pi}\, T^{(\pi\to F)} \,,$$

$$b_i'^A \frac{g_{\ell A_1}}{q^2+m_A^2} = b_i^A \,, \text{ as defined in (18)} \,, \tag{25}$$

where b_0^A is proportional to the contribution of $\{F\}$ to the absorptive part of the amplitude $\pi\to A_{1L}$; let us call it abs $F^{(\pi\to A1L)}$.

(c) Diagonal with respect to the π

$$q_\mu q_\nu \frac{g_{\ell\pi}^2}{(q^2+m_\pi^2)} |\boldsymbol{q}|\, \sigma^{(\pi\to F)} \,.$$

We easily verify that the $q_\mu q_\nu$ term proportional to $b_1^A T^{(\pi\to F)}/(q^2+m_\pi^2)(q^2+m_A^2)$ (see eq. (2) gives negligible contribution with respect to the one proportional to $|T^{(\pi\to F)}|^2/(q^2+m_\pi^2)^2$ (sect. 4). (The order of the neglected terms is m^2/m_A^2 and q^2/m_A^2, q^2 being of few m_π^2.)

Let us briefly express in the framework of that model the general results obtained in the preceding section.

The constraint of PCAC implies relations valid near $q^2\sim 0$ between the scattering of the longitudinal A_1 and of the π, more precisely

$$\frac{g_{\ell A_1}^2}{m_A^4} \lim_{q^2\to 0} |q^2|\, \sigma_L^{(A1\to F)}(|q^2|) \to g_{\ell\pi}^2\, \sigma^{(\pi\to F)}(|q^2|) \,, \tag{26}$$

see eqs. (20) and (24), and

$$\frac{g_{\ell A_1}}{m_A^2} \lim_{q^2\to 0} \frac{\sqrt{|q^2|}}{M|\boldsymbol{q}|} \text{abs}\, F^{(\pi\to A1L)}(|q^2|) \to g_{\ell\pi}\, \sigma^{(\pi\to F)}(|q^2|) \,. \tag{27}$$

In other words, the scattering of a longitudinal A_1 near $q^2\sim 0$ acts like the π-scattering. We shall see later on that if we attempt to apply this to the physical particles, the result disagrees with experiment.

Relations (26) and (27) assure that we get the generalized Adler's relation as given by (21) for ν-scattering near $q^2\sim 0$.

At large q^2, in the region $q^2 \gg m_\pi^2$ we have for the axial contribution to the weak production of {F} an expression close to that of the vector (see (11)) replacing the quantities relative to the ρ by the corresponding A_1 quantities:

$$\frac{d^2\sigma(\begin{smallmatrix}\nu\\ \bar\nu\end{smallmatrix}\to F)(A)}{dq^2\,d\nu} = \frac{G^2}{\pi^2}\frac{1}{4E^2}\,g^2_{\ell A_1}\,\frac{q^2}{(q^2+m^2_A)^2}\,|\boldsymbol{q}|\,[\sigma_T^{(A_1^{\pm}\to F)} + \frac{(\sigma_T+\sigma_L)^{(A_1^{\pm}\to F)}}{2|\boldsymbol{q}|^2}\,(4EE'-q^2)]\,. \tag{28}$$

2.3. *Interference between axial and vector currents*

Let us consider the vector, axial interference term. From Lorentz invariance and parity of (A · V) equal to -1, one verifies that the hadronic tensor:

$$\bar{\sum_I}\sum_F A_\mu^{*I\to F}\,V_\nu^{I\to F} \quad \text{projects only on} \quad \epsilon_{\mu\nu\lambda\rho}P_\lambda q_\rho\,:$$

$$\frac{G_1(q^2,\nu)}{M^2}\,\epsilon_{\mu\nu\lambda\rho}P_\lambda q_\rho = \bar{\sum_I}\sum_F A_\mu^{*I\to F}\,V_\nu^{I\to F}\,. \tag{29}$$

Straightforward calculations lead to the following contribution to the ν or $\bar\nu$ scattering.

$$\frac{d^2\sigma(A\times V)}{dq^2\,d\nu} = \pm\frac{G^2}{2\pi^2}\frac{1}{ME}\,G_1 q^2\,. \tag{30}$$

(i) Meson dominance. In the framework of the meson-dominance model, this means that neither the π nor the longitudinal A_1 contribute to the interference, when $\bar\Sigma_I\,\Sigma_F$ is over polarization of the target and over all momenta and spins of {F}, more precisely:

$$\frac{G_1|\boldsymbol{q}|}{M} = \frac{g_{\ell A_1}}{q^2+m^2_A}\,\frac{g_{\ell\rho}}{q^2+m^2_\rho}\,[\bar{\sum_I}\sum_F T_\nu^{*(A_1\to F)}\cdot\epsilon_\nu^{*T}(A_1)\,T_\mu^{(\rho\to F)}\cdot\epsilon_\mu^{T}(\rho)]\,, \tag{31}$$

where the quantity in [ ] can be identified with $(1/2M)$ abs $F^{(\rho T\to A1T)}$, that is the contribution of {F} to the absorptive part of the $\rho_T\to A_{1T}$ amplitude. An upper limit for it can be obtained.

(ii) Generalized Schwartz inequality - upper limit for the vector axial currents interference contribution. The right-hand side of (29) can be written as

$$\sum_F\langle\mathcal{N}|A_\nu^{*T}\cdot\epsilon_\nu^{*T}|F\rangle\langle F|V_\mu\cdot\epsilon_\mu^{T}|\mathcal{N}\rangle\,,$$

where average over the target spin is implied.

This expression looks like a generalized scalar product of two 'vectors' (in the Dirac bra and ket sense) namely $V_\mu \cdot \epsilon_\mu^{\rm T} |\mathcal{N}\rangle$ and $A_\nu \cdot \epsilon_\nu^{\rm T} |\mathcal{N}\rangle$ and using exactly the same trick as for the usual Schwartz inequality, one derives:

$$\sum_{\rm F} \langle \mathcal{N} | A_\nu^* \cdot \epsilon_\nu^{T*} | {\rm F}\rangle \langle {\rm F} | V_\mu \cdot \epsilon_\mu^{\rm T} | \mathcal{N}\rangle | \leq \sqrt{\sum_{\rm F} |\langle {\rm F} | A_\nu \cdot \epsilon_\nu^{\rm T} | \mathcal{N}\rangle|^2 \sum_{\rm F} |\langle {\rm F} | V_\mu \cdot \epsilon_\mu^{\rm T} | \mathcal{N}\rangle|^2} \,. \quad (32)$$

In terms of the form factor already defined (in (1), (13), (29)) it implies:

$$|G_1(q^2, \nu)| \frac{|q|}{M} \leq \sqrt{F_0^{\rm V}(q^2,\nu) F_0^{\rm A}(q^2,\nu)} \,. \quad (33)$$

We can note an amusing consequence of (33), that is if the F_0 values and G_1 satisfy dispersion relations at fixed q^2, G_1 requires one subtraction less than the F_0 values †.

In the framework of the meson dominance, inequality (33) takes the following form:

$$|{\rm abs\, F}^{(\rho_{\rm T} \to {\rm A}_{1\rm T})}| \leq \sqrt{|{\rm abs\, F}^{(\rho_{\rm T} \to \rho_{\rm T})}| \, |{\rm abs\, F}^{({\rm A}_{1\rm T} \to {\rm A}_{1\rm T})}|} \; ^{\ddagger} \,. \quad (34)$$

When the sum over F is over all available states abs $F^{(\rho_{\rm T} \to {\rm A}_{1\rm T})}$ corresponds to the imaginary part of the $\rho_{\rm T} \to {\rm A}_{1\rm T}$ amplitude by unitarity condition.

This leads to an upper limit for the modulus of the vector axial, interference term:

$$\frac{{\rm d}^2\sigma^{(\frac{\nu}{\bar\nu} \to {\rm F})}}{{\rm d}q^2\, {\rm d}\nu} ({\rm A}\cdot{\rm V}) \leq \frac{G^2}{\pi^2} \frac{1}{E} \frac{g_{\ell {\rm A}_1} g_{\ell \rho}}{(q^2 + m_{\rm A}^2)(q^2 + m_\rho^2)} q^2 \sqrt{\sigma_{\rm T}^{(\rho^\pm \to {\rm F})} \sigma_{\rm T}^{({\rm A}^\pm \to {\rm F})}} \,, \quad (35)$$

where the ± sign refers to ν and $\bar\nu$ scattering.

For q^2 *close to zero* we have for the production of $\{{\rm F}\}$ the generalized Adler's formula (21); and for $q^2 \gg m_\pi^2$

† Let us remark, although it is outside the framework of this paper, that similar inequalities can be deduced for other processes such as ρ production by π (using $\partial_\mu A_\mu$ instead of $A_\mu \cdot \epsilon_\mu^{\rm T}$ in (32)) or for vertex functions (if one of the $\mathcal{N}$ state in (32) is replaced by the hadronic vacuum); or for form factors with state $|\mathcal{N}\rangle$ in a definite polarization state i (taking for example the vectors $\epsilon_\nu^\alpha \cdot V_\nu |\mathcal{N}_i\rangle$ and $\epsilon_\mu^\beta \cdot V_\mu |\mathcal{N}_j\rangle$) etc...

‡ Similar inequalities have been already obtained in ref. [8]. We wish to thank J. L. Gervais for bringing this work to our notice.

$$\frac{d\sigma^{(\nu\to F)}}{dq^2\,d\nu} = \frac{G^2}{\pi^2}\frac{1}{4E^2}\Big[|\boldsymbol{q}|\Big\{g^2_{\ell\rho}\frac{q^2}{(q^2+m_\rho^2)^2}\Big[\sigma_T^{(\rho\to F)} + \frac{\sigma_T^{(\rho\to F)}+\sigma_L^{(\rho\to F)}}{2|\boldsymbol{q}|^2}(4EE'-q^2)\Big]$$

$$+g^2_{\ell A_1}\frac{q^2}{(q^2+m_A^2)^2}\Big[\sigma_T^{(A_1\to F)} + \frac{\sigma_T^{(A_1\to F)}+\sigma_L^{(A_1\to F)}}{2|\boldsymbol{q}|^2}(4EE'-q^2)\Big]\Big\}$$

$$\pm\frac{4g_{\ell\rho}\,g_{\ell A_1}}{(q^2+m_A^2)(q^2+m_\rho^2)}\,q^2E\,\sqrt{\sigma_T^{(\rho\to F)}\,\sigma_T^{(A_1\to F)}}\Big]\,. \qquad (36)$$

3. APPLICATIONS OF THE MODEL: ASSUMPTIONS REQUIRED - ESTIMATES OF THE COUPLING CONSTANTS AND UNPHYSICAL MESON-NUCLEON SCATTERING CROSS SECTIONS

3.1. *Symmetry* ρ - A_1

We shall treat the A_1 meson as the chiral partner of the ρ. Let us recall that this assumption has received some support from the successful calculations of $\pi^+\pi^0$ mass difference and of the ratio of the A_1 to the ρ masses derived from spectral sum rules [9]. Roughly speaking this implies that we shall assume the unknown measured quantities for the A_1 equal to that of the corresponding ones for the ρ, mainly: $g_{\ell A_1} = g_{\ell\rho}$ and A_1 scattering cross sections about the same as those for the ρ. We shall come back later to the problem of the longitudinal A_1.

3.2. *Coupling constants*

The three basic constants $g_{\ell\rho}$, $g_{\ell A_1}$ and $g_{\ell\pi}$, giving the coupling of the mesons to the leptonic weak current, play the role of the $\gamma\rho^0$ coupling in electromagnetic interactions, and are in principle measured by the decays ρ^+, A_1^+, $\pi^+\to\mu^+\nu$, given by effective couplings

$$\frac{G}{\sqrt{2}}\,\hat{\mu}\gamma_\mu(1+\gamma_5)\nu\begin{pmatrix} g_{\ell\rho}\,\rho_\mu \\ g_{\ell A_1}\,A_{1\mu} \\ g_{\ell\pi}\,\partial_\mu\varphi \end{pmatrix}\,,$$

just as $g_{\gamma\rho^0}$ is given by $\rho^0\to e^+e^-$. While $g_{\ell\pi}$ is known directly in this manner: $g_{\ell\pi}\simeq 0.93\,m_\pi$, $g_{\ell\rho}$ may be inferred (from the measurement of $\rho^0\to e^+e^-$) by CVC, or more simply by using the fact that ρ-dominance must give the correct nucleon β-decay constant (cf. fig. 4).

Let f_ρ be the universal ρ coupling constant ($f_\rho^2/4\pi = 2.1$) (ref. [2]) coupling the ρ (to N, π . . .) like:

$$f_\rho\,\rho_\mu[\bar{\Psi}\gamma_\mu\tfrac{1}{2}\tau\,\psi + \varphi\times\partial_\mu\varphi + \ldots]\,,$$

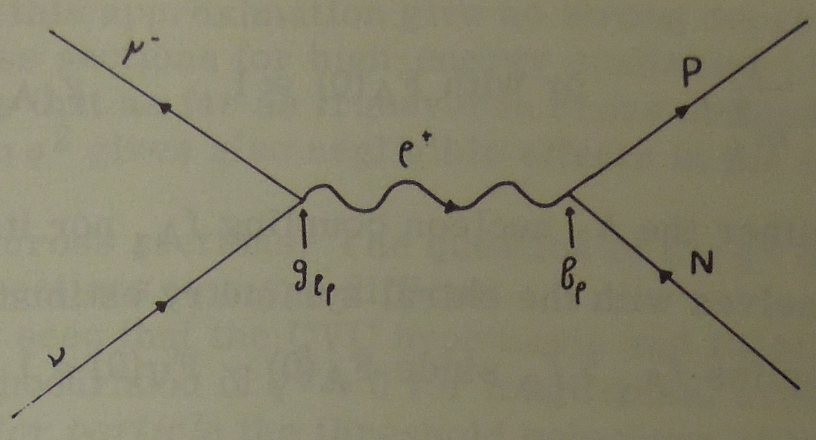

Fig. 4. ρ-dominance in nucleon β-decay.

then, at zero-momentum transfer in fig. 4, we have for the vector part of the hadronic current:

$$\frac{G}{\sqrt{2}}\,\bar{\ell}\gamma_\mu(1+\gamma_5)\nu\bar{P}\gamma_\mu N = \frac{G}{\sqrt{2}}\,\bar{\ell}\gamma_\mu(1+\gamma_5)\nu\frac{g_{\ell\rho}}{m_\rho^2}\frac{f_\rho}{\sqrt{2}}\,\bar{P}\gamma_\mu N\,,$$

so that

$$g_{\ell\rho} = \frac{\sqrt{2}}{f_\rho}\,m_\rho^2 \qquad \left(\text{to be compared with} \quad g_{\gamma\rho^0} = \frac{m_\rho^2}{f_\rho}\right). \tag{37}$$

We can repeat the above argument for the axial current: the A_1 and π dominance for the axial current must give the correct axial nucleon current, (with f_{A_1}, f_π the couplings to the nucleon)

$$\frac{G}{\sqrt{2}}\,\bar{\ell}\gamma_\mu(1+\gamma_5)\nu[\bar{P}\gamma_\mu\gamma_5 N\, F_A + iF_P\, q_\mu\, \bar{P}\gamma_5 N]$$

$$= \frac{G}{\sqrt{2}}\,\bar{\ell}\gamma_\mu(1+\gamma_5)\nu[g_{\ell A_1}\frac{(\delta_{\mu\lambda}+q_\mu q_\lambda/m_A^2)}{q^2+m_A^2}\,f_{A_1}\frac{1}{\sqrt{2}}\,\bar{P}\gamma_\lambda\gamma_5 N$$

$$+\, g_{\ell\pi}\frac{iq_\mu}{q^2+m_\pi^2}\sqrt{2}\,f_\pi\bar{P}\gamma_5 N]\,.$$

One sees that in the model the induced pseudoscalar term F_P contains besides the ordinary π-pole term a part coming from the longitudinal A_1, namely:

$$F_P \sim \frac{g_{\ell A_1}}{m_A^2}\frac{2M}{q^2+m_A^2}\frac{f_{A1}}{\sqrt{2}} + g_{\ell\pi}\frac{\sqrt{2}}{q^2+m_\pi^2}f_\pi\,.$$

At low momentum transfer we must have:

$$F_A(0) = \frac{f_{A_1}}{\sqrt{2}} g_{\ell A_1} \frac{1}{m_A^2}, \qquad \text{or with } F_A(0) \cong 1, \qquad g_{\ell A_1} \cong \frac{\sqrt{2}\, m_A^2}{f_{A_1}}. \qquad (37')$$

Since we know neither the A_1 nucleon coupling f_{A_1} nor its weak decay, we must content ourselves with the chiral symmetry estimate $g_{\ell A_1} \sim g_{\ell\rho}$ †. (Note that this implies $f_{A_1} > f_\rho$ since $F_A(0) \sim F_V(0) = 1$ (see (37) and (37')).

3.3. *Unphysical meson scattering cross sections*

The cross sections for unphysical incident mesons introduced in the preceding section depend on q^2 the $(\text{mass})^2$ of the meson, and on its energy. The question arises: can we use the existing data for real meson scattering? Of course, for weak vector production, when they are available, we use electroproduction data that are already for an unphysical ρ-meson.

(i) Energy dependence. As for the energy dependence in the high-energy limit, we can use the same dependence as for physical meson scattering when for high-energy particle scattering the mass can be neglected with respect to the energy ‡. In other words the model is restricted to ν large relative to $\sqrt{q^2}$. In practice this implies that we take as constant with ν the total and elastic cross sections for ρ, A_1 and π-scattering, as well as the cross sections for diffraction like processes such as longitudinal A_1 production by unphysical π (or the inverse, production of π by unphysical A_{1L}). On the other hand for the production of ρ by unphysical π (or the inverse process) we use the $1/\nu^2$ dependence of the cross section for the physical processes $\pi \to \rho$ on a nucleon. In that case where the one π exchange is dominant, one verifies by direct calculation that the energy dependence remains the same for the physical processes and unphysical ones [11].

(ii) Extrapolation of q^2. To answer that question one must study separately the transverse and longitudinal cross sections.

(a) Transverse cross sections. As far as the transverse cross section is concerned, there are some indications that the extrapolation in q^2 is smooth. First, as we have already pointed out in the introduction, the success of ρ-dominance for the interpretation of photoproduction experiments indicates that the necessary extrapolation from the physical ρ-meson mass to the zero mass of the photon gives almost no effect for the transverse meson, which is the only one to be coupled to the photon. Moreover, in the particular case where one-particle-exchange should be dominant, direct

† This relation corresponds to the second Weinberg sum rule and is in agreement with the first one using $m_{A_1} = \sqrt{2}\, m_\rho$ (ref. [10]).

‡ Let us remark that the natural quantities to be extrapolated from physical scattering to unphysical scattering are the transition rates rather than the cross sections. However, in the high-energy limit, we consider, this makes almost no difference. The difference lies on the rates of the flux factors (cf. (7)) namely $\sqrt{(\nu^2-m^2)(\nu^2+q^2)}$ which is of the order of 1 for $\nu \gg \sqrt{q^2}$ and for simplicity we omit this pure kinematic factor that can be easily taken into account.

calculations within this approximation give no strong dependence on q^2 for the transverse cross sections for high-energy scattering [11]. So it seems plausible to assume that as far as transverse cross sections are concerned, the extrapolation on q^2 gives also negligible effects in the space-like region we consider.

(b) Longitudinal cross sections. The question of the q^2 dependence of the longitudinal cross sections is more difficult.

We have already seen that the CVC hypothesis and PCAC impose constraints on the neighbourhood of $q^2 = 0$ for longitudinal cross sections. In particular, for *vector particle* the threshold behaviour of longitudinal cross sections must be as q^2, showing a strong dependence on the unphysical ρ-meson mass, at least near $q^2 = 0$.

We may remark that for a non-diffractive process such as π-production, explicit calculations with simple exchanges, show that longitudinal amplitudes are proportional to $\sqrt{q^2}$ (ref. [11]).

For the axial current, PCAC relates longitudinal A_1 scattering to the π-amplitude (see (26)-(27)), i.e. in terms of the model: the scattering of a longitudinal A_1 at q^2 equal zero, acts like π-scattering.

Let us note that the relation coming from PCAC does not experimentally hold at all for physical longitudinal A_1. Let us try to apply (26) at $|q^2| = m_A^2$, assuming that the invariant functions $F^A(q^2, \nu)$ (cf. (13)), are almost constant. Experimentally, we know the elastic scattering cross section of π on protons, that is, $\sigma^{(\pi\to\pi)}(|q^2| = m_\pi^2)$, and the production cross section of longitudinal A_1 by incident π on protons, that is: $\sigma^{(\pi\to A_{1L})}(|q^2| = m_A^2)$. Both are diffractive like at high energy so that these cross sections appear to tend to constants respectively 4.2 and 0.2 mb (ref. [12]). For the true elastic cross section $\sigma^{(\pi\to\pi)}$, the extrapolation on the mass squared is expected to give small effect, so that we assume $\sigma^{(\pi\to\pi)}(|q^2| = m_A^2) \simeq \simeq \sigma^{(\pi\to\pi)}(|q^2| = m_\pi^2)$. If we assume moreover that $\sigma^{(\pi\to A_{1L})}(|q^2| = m_A^2)$ is equal to the inverse process $\sigma^{(A_{1L}\to\pi)}(|q^2| = m_A^2)$ we might try in the high-energy limit:

$$\frac{\sigma^{(A_{1L}\to\pi)}(|q^2| = m_A^2)}{\sigma^{(\pi\to\pi)}(|q^2| = m_A^2)} \simeq \frac{\sigma^{(\pi\to A_{1L})}}{\sigma^{(\pi\to\pi)}_{\exp}} = \frac{1}{20},$$

while the theoretical ratio deduced from (26) making use of the $g_{\ell\pi}$ and $g_{\ell A_1}$ estimates, turns out to be equal to 1. So that the extrapolation for $|q^2| \gg m_\pi^2$ of the estimate of the longitudinal A_1 scattering cross sections deduced from PCAC seems highly dubious.

In practice, in both vector and axial cases, we can guess little about the behaviour of σ_L away from $q^2 \simeq 0$ and in fact, two extreme points of view can be adopted:

The first one is to assume that the constraint of CVC and PCAC remain valid away from $q^2 \sim 0$, that is for vector-meson scattering: $\sigma_L^{(\rho)}(q^2)$ remains of the order of zero, or eventually keeps the threshold linear q^2 dependence:

$$\sigma_L^{(\rho)}(q^2) = \frac{|q^2|}{m_\rho^2}\,\sigma_L^{(\rho)}(m_\rho^2)\,, \tag{38}$$

in order to get the correct normalization for the physical ρ-meson †; for axial-meson scattering, this corresponds to keep, for large q^2:

$$|q^2|\,\sigma_L^{(A_1\to F)}(|q^2|) \sim g_{\ell\pi}\,2\,\frac{m_A^4}{g_A^2}\,\sigma^{(\pi\to F)}(|q^2|)\,.$$

The second one, that is in fact the most naive ρ-dominance, is to remark that in the high-energy limit, where diffraction is dominant, the corresponding cross section would be independent of polarization, so that for diffractive process σ_L should be taken as equal to σ_T away from $q^2 = 0$, assuming that the constraints of CVC or PCAC lead to some threshold structure. In fact the behaviour of longitudinal cross section away from very small q^2 is highly uncertain. We will use explicitly these two kinds of behaviour in order to see the kind of uncertainty of our results coming from the ambiguity on the longitudinal cross sections.

4. RESULTS

4.1. *Electroproduction*

(i) Total electroproduction cross sections and the plausibility of the model. In order to check the validity of the model, let us now compare the results obtained with the experimental data on deep electroproduction [3] total cross sections, by which we mean the cross sections summed over all final hadronic states for a given q^2, ν. These data are interpreted in terms of the W_1 and W_2 structure functions defined in sect. 1, eqs. (3) and (10). In practice, keeping the scattering angle very small, $W_2(q^2, \nu)$ was picked out and its variations with q^2 and ν studied. For clarity, let us recall the form of W_2 (see eq. (10)), keeping the ω and ρ^0 contributions:

$$W_2(q^2,\nu) = \frac{g_{\gamma\rho^0}^2}{\pi}\,\frac{(\sigma_T+\sigma_L)_{\rm tot}^{(\rho)}}{m_\rho^4}\,\frac{q^2}{|\boldsymbol{q}|(1+q^2/m_\rho^2)^2} + \frac{g_{\gamma\omega}^2}{\pi}\,\frac{(\sigma_T+\sigma_L)_{\rm tot}^{(\omega)}}{m_\omega^4}\,\frac{q^2}{|\boldsymbol{q}|(1+q^2/m_\omega^2)^2}\,. \tag{10'}$$

† This is precisely the hypothesis used in refs. [13,14] for total cross section with $\sigma_L(m_\rho^2) = \sigma_T(m_\rho^2) \simeq \sigma_T(q^2)$.

The data on W_2 show that at large ν (or $|\boldsymbol{q}|$) for fixed q^2, W_2 drops slowly, perhaps as $1/\nu$. This is expected in our diffractive approach from (10') since σ_{tot} should be constant with ν. As for the q^2 dependence, if we take the naive ρ-dominance, so that the ρP scattering is assumed to be q^2 independent, we get:

$$W_2 \sim \frac{q^2}{(1+q^2/m_\rho^2)^2}\,.$$

The variation is quite consistent with experimental results which cover a relatively narrow q^2 range at large ν. The absolute magnitude for W_2 at large ν predicted in this way, however from σ_T alone, is about a factor of two too small. It seems reasonable to argue that the difference is due to the σ_L contribution and implies that for $q^2 \neq 0$, $\sigma_L(q^2)$ is non-zero. If we keep the naive idea that diffractive ρP scattering should be independent of polarization when q^2 is not too small, we have $\sigma_L \sim \sigma_T$. Neglecting the φ-contribution from total photoproduction, we deduce:

$$\frac{g^2_{\gamma\omega}}{m^4_\omega}\,\sigma_T^{(\omega)} + \frac{g^2_{\gamma\rho^0}}{m^4_\rho}\,\sigma_T^{(\rho^0)} = \frac{\sigma_{tot}(\text{real }\gamma)}{4\pi\alpha}\,. \qquad (39)$$

Within these approximations and neglecting the small $\omega\rho$ mass difference, $W_2(q^2,\nu)$ takes the following form:

$$W_2(q^2,\nu) \simeq \frac{\sigma(\text{real }\gamma)(1+R)}{4\pi^2\alpha}\;\frac{q^2}{|\boldsymbol{q}|(1+q^2/m_\rho^2)^2}\,, \qquad (40)$$

where R is the ratio of longitudinal to transverse contributions that we take equal to 1.

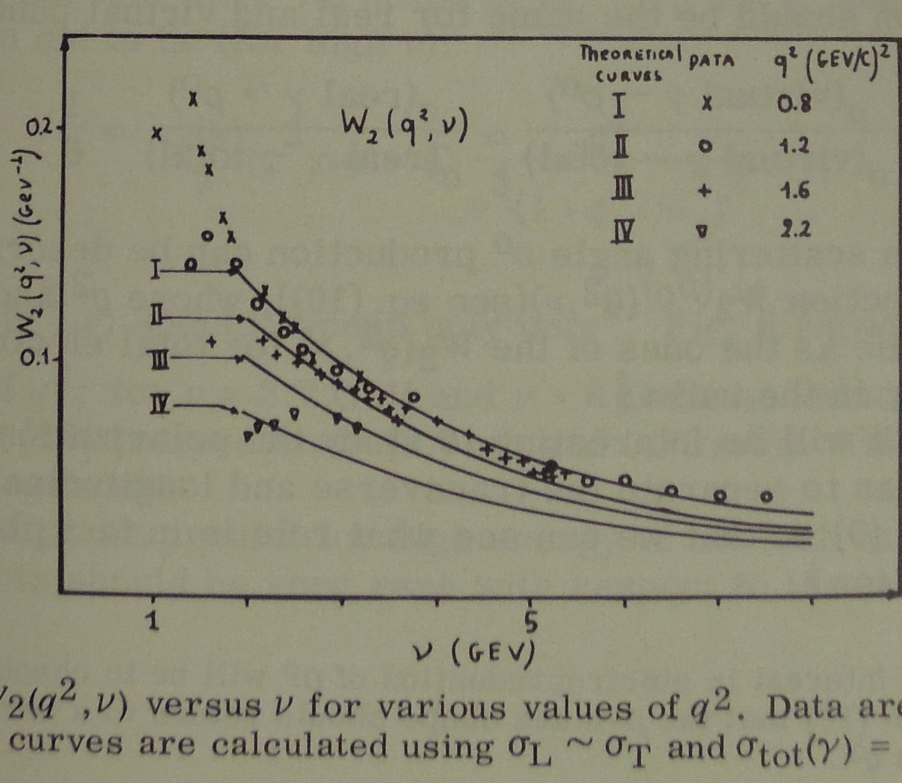

Fig. 5. Plot of $W_2(q^2,\nu)$ versus ν for various values of q^2. Data are taken from ref. [3]. Theoretical curves are calculated using $\sigma_L \sim \sigma_T$ and $\sigma_{tot}(\gamma) = 120\,\mu$B (ref. [1]).

Results are illustrated on fig. 5. We have plotted $W_2(q^2, \nu)$ versus ν for various values of q^2 (q^2 = 0.8, 1.2, 1.6, 2.2 $(\mathrm{GeV}/c)^2$). The theoretical curves are calculated from (40) using the measured value of the total photoproduction cross section of 120 μb (ref. [2]), and assuming $\sigma_L \sim \sigma_T$, that is $R = 1$. The data are taken from ref. [3]. We see that the theoretical curves and the high-energy electroproduction data are roughly in agreement. This indicates that the naive diffractive ρ-dominance, that is the total cross section at high energy independent of both mass and polarization of the unphysical ρ-meson is quite plausible. A second possibility entertained for W_2 is that it becomes independent of q^2 (in our notation $(q^2 m_v^4/(q^2 + m_v^2)^2)\, \sigma^v(q^2) \sim$ constant), and is also roughly consistent with the data available (this is due to the small domain of measured q^2). This absence of structure for W_2 is predicted in the framework of the Feynman's model of the nucleon [15] where at large-momentum transfer and high energy, the inelastic scattering is pictured as quasi-free scattering from point-like constituents (partons) within the proton [16]. It may be pointed out that in the limit of high q^2 the same kind of behaviour for W_2 namely $W_2 \to$ constant is also obtained in the framework of ρ-dominance if one keeps the threshold linear q^2 dependence for high q^2 (see (38)); this is precisely the assumption of Sakurai [13], in that case, for large q^2, the scattering of longitudinal ρ-meson gives the main contribution to deep electroproduction.

Anyway it will be important to extend the domain of measured q^2 in electroproduction experiment so as to see whether or not W_2 decreases with q^2.

In conclusion, although we cannot calculate with confidence the absolute magnitude of W_2, the experimental data are consistent with the diffraction view-point in the sense that at high energy the diffraction-like cross sections may become independent of the polarization and mass of the incident virtual meson.

(ii) ρ^0 electroproduction. No experimental data on ρ^0 electroproduction are available. However, we can estimate the ρ^0 electroproduction cross section assuming that the ratio of ρ^0 production cross section to total cross section by photon should be the same for real and virtual photon, that is:

$$\frac{\sigma(\text{virtual } \gamma \to \rho^0)}{\sigma(\text{virtual } \gamma \to \text{total})} \simeq \frac{\sigma(\text{real } \gamma \to \rho^0)}{\sigma(\text{real } \gamma \to \text{total})} \simeq \frac{1}{6}\,. \tag{41}$$

The small lepton scattering angle ρ^0 production can be described in terms of the structure function $W_2^{(\rho^0)}(q^2, \nu)$ (see eq. (10)), whose q^2 and ν behaviours would be the same as the ones of the $W_2(q^2, \nu)$ for total electroproduction (10') and roughly in the ratio $\frac{1}{6}$.

We note that it will be interesting to study the polarization of the produced ρ as well as to separate the transverse and longitudinal contributions to cross section (9) so that we can see what role is in fact played by longitudinal polarization †.

† Another point of interest in electroproduction of ρ^0 will be to check the Ross-Stodolsky prediction [17] that the ρ-mass shape should return to a normal Breit-Wigner form at high q^2.

(iii) $\pi^{\pm}$ electroproduction. Finally we calculate the high-energy electroproduction of charged π. The leading contribution comes from the longitudinal ρ, so that one has for high energy of the $\pi\mathcal{N}$ system and moderate scattering angle:

$$\frac{d^2\sigma^{(e\to\pi)}}{d\Omega dE'} \sim \frac{\alpha^2}{4E^2}\frac{1}{\sin^4(\frac{1}{2}\theta)} W_2^{(\pi)} \cos^2(\tfrac{1}{2}\theta) ,$$

with

$$W_2^{(\pi)}(q^2,\nu) \simeq \frac{g^2_{\gamma\rho^0}}{\pi}\frac{\sigma_L^{(\rho^0\to\pi)}(q^2,\nu)}{m_\rho^4}\frac{q^2}{|\boldsymbol{q}|(1+q^2/m_\rho^2)^2} . \tag{42}$$

(where we have neglected in (3) and (10) the transverse ρ contribution). We use the experimental data in $\sigma(\pi - \rho_L)$ (ref. [18]) and keep in the unphysical region for the ρ the same variation with the incident meson energy: $1/p_0^2$ and assume a linear variation with the ρ-meson mass as is suggested by simple one-π-exchange model (see sect. 3 and ref. [11]). As for the form factor of the π exchanged, the model corresponds to assuming a simple ρ-dominance pion form factor.

$$\sigma_L^{(\rho\to\pi)}(q^2,\nu) \simeq \frac{q^2}{m_\rho^2}\sigma_L^{(\pi\to\rho)}(m_\rho^2, p_0=\nu) ,$$

so that

$$W_2^{(\pi)}(q^2,\nu) \simeq \frac{g^2_{\gamma\rho^0}}{\pi m_\rho^4}\frac{q^2}{m_\rho^2}\sigma_L^{(\pi\to\rho)}(m_\rho^2,\nu)\frac{q^2}{|\boldsymbol{q}|(1+q^2/m_\rho^2)^2} ,$$

and taking into account the ν variation of $\sigma_L^{(\pi\to\rho)}$, the q^2 and ν behaviour of $W_2^{(\pi)}(q^2,\nu)$ turn out to be (for high ν):

$$W_2^{(\pi)}(q^2,\nu) \sim C\frac{q^4}{\nu^3(1+q^2/m_\rho^2)^2} ,$$

so that $\nu^3 W_2^{(\pi)}(q^2,\nu)$ should depend only on q^2. Fig. 6 shows $\nu^3 W_2^{(\pi)}(q^2,\nu)$ as a function of q^2; for $\nu = 2.7$ GeV and $\nu = 8$ GeV the values of $\sigma^{(\pi\to\rho_L)}$ are taken from [18] (respectively (2.7 mb and 0.23 mb)). The two curves cannot be distinguished.

Let us note that we predict for high energy of the hadronic state that the one pionic events should be very rare with respect to the two pionic ones.

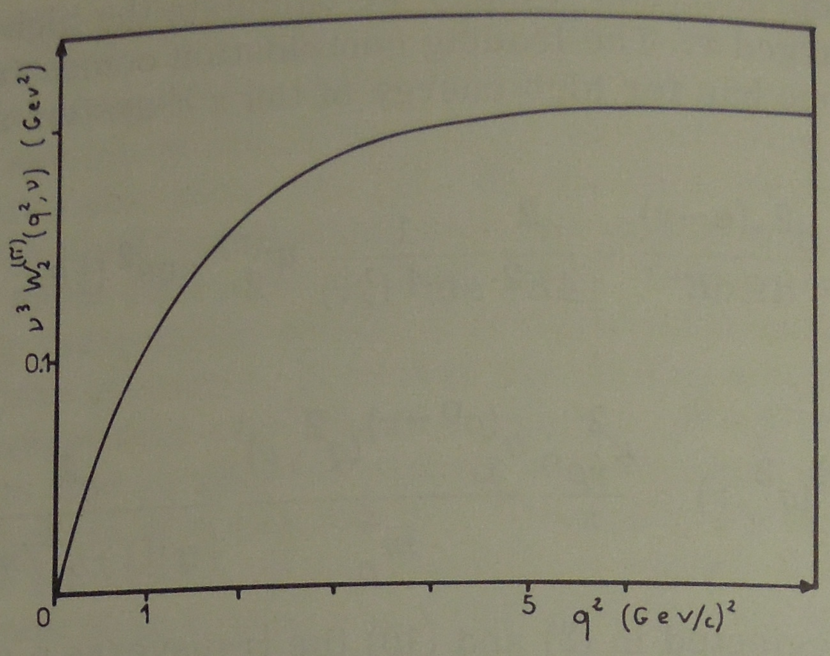

Fig. 6. Plot of $\nu^3 W_2^{(\pi)}(q^2,\nu)$ for π-electroproduction versus q^2 for ν equal 2.7 GeV and 8 GeV. Data for $\sigma^{(\pi\to\rho_L)}$ are taken from ref. [18]. The two curves cannot be distinguished.

4.2. *Weak production*

(i) ρ-production

(a) Dominant elastic contribution. If we consider a final state $\{F\}$ like ρP where the vector current should dominate, the cross section should be given simply by (11). Moreover the diffraction character makes it plausible that charged and neutral ρ-scattering are about the same, then we can have a certain degree of model independence while still testing the basic diffraction assumption by relating neutrino ρ-production directly to electroproduction by way of (see eqs. (10, (11) and (12))

$$\frac{d^2\sigma^{\binom{\nu\to\rho^+}{\bar{\nu}\to\rho^-}}}{dq^2\,d\nu} = 4q^4\,\frac{G^2}{e^4}\,\frac{d^2\;(e\to\rho^0)}{dq^2\,d\nu}\,. \tag{43}$$

Let us remark that more generally, similar formulae could be used to estimate the neutrino production of a system $\{F\}$ when the vector current gives the main contribution.

No experimental results on ρ^0 electroproduction are available but we have already estimated ρ^0 electroproduction using data on total electroproduction and photoproduction (see (41)). In practice we assume $\sigma_L^{(\rho\to\rho)}$ and $\sigma_T^{(\rho\to\rho)}$ to be equal to 5.2 mb. Fig. 7 curve I shows for an incident ν energy of 3 GeV the momentum distribution of the ρ-production. We have verified that to take into account the increase of $\sigma_{tot}^{(\gamma\to\rho^0)}$ for small ν (ref. [19]) makes a change by no more than 10% on the maximum transverse contribution to $(d\sigma/dq^2)^{(\nu\to\rho_T)}$.

In fact the total elastic cross section for unphysical ρ-meson should be smaller than the one for real scattering. This comes from the fact that the

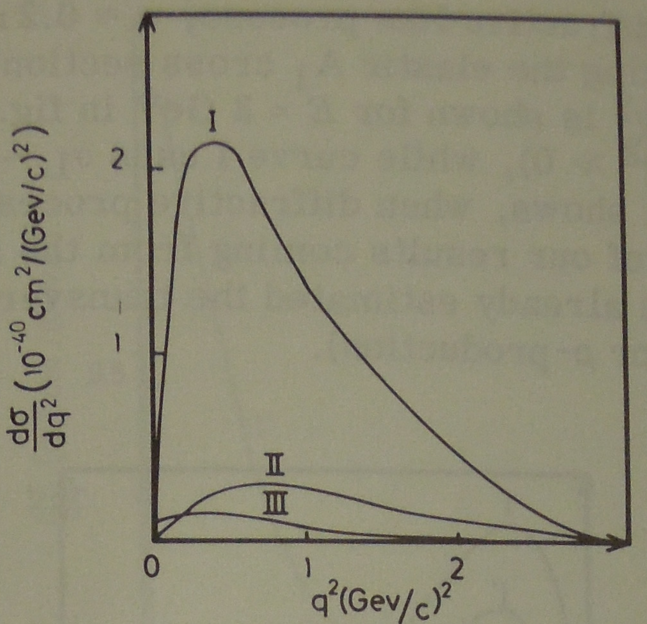

Fig. 7. q^2 distribution for ρ production by incident neutrino of 3 GeV. Curve I shows the diffractive-like ρ-contribution calculated using electroproduction data ($\sigma_L \sim \sigma_T$). Curve II shows an upper limit for the interference $A_1 \rightarrow \rho$ transverse contribution. Curve III shows the $\pi \rightarrow \rho_L$ contribution using (26).

minimum momentum transfer to the nucleon (let us call it t_m) is no longer zero. t_m depends on q^2 and ν, the (mass)^2 and energy of the incoming meson and is approximately equal for high energy to $((q^2 + m^2)/2\nu)^2$. The off-mass-shell effects † can be roughly taken into account by a damping factor e^{-at_m} (for $\nu < 5$ GeV, $a \approx 5(\text{GeV}/c)^{-2}$ (ref. [21]). With these hypotheses, we find that the main effect in curve I can be reduced to a damping factor of the order of 0.65.

We note of course that in neutrino production it will be useful too, to study the polarization of the produced ρ in order to see what role is in fact played by a longitudinal polarization.

(b) Non-diffractive contributions. Curve II gives an upper bound on the A_{1T} (i.e. axial) contribution to ρ-production if it is limited by the $\sigma(\rho - A_{1T})$ limit inferred from charged A_1 photo-production data (namely < 0.1 mb for $\nu > 2$ GeV). For a fixed incident neutrino energy a non-diffractive contribution like this drops with the incident meson energy (perhaps like $1/\nu^2$ as suggested by naive π-exchange) and one verifies that with increasing neutrino energy it becomes more and more negligible with respect to the dominant diffractive contributions. As for the contribution coming from π and longitudinal A_1, we have used the measured ρ-production cross section by π and moreover assumed for the A_{1L} the validity of (26) even for q^2 larger than few m_π^2. The resulting contribution turns out to be completely negligible with respect to the diffractive one, even for small incident neutrino energy and relatively small momentum transfer. (Of course, at $q^2 = 0$, where the vector part is strictly zero by CVC, the axial current is dominant but turns out to be quite small.)

(ii) A_1 production. To estimate A_1 production by the axial current we have (21) for small q^2 with $\sigma(\pi P \rightarrow A_1 P)$ known from experiment, an ap-

† Effects of this type were taken into account by Roe [20].

proximately constant diffractive-like process, $\sigma \approx 0.2$ mb. Away from $q^2 \approx 0$ we use (36), taking the elastic A_1 cross section the same as for the ρ. The result for $d\sigma/dq^2$ is shown for $E = 3$ GeV in fig. 8, where curve I', using (26) for σ_L (for $q^2 \gg 0$), while curve I uses $\sigma_L \sim \sigma_T$. The difference between curves I and I' shows, when diffractive processes are dominant, the kind of uncertainty of our results coming from the ambiguity on the longitudinal part. We have already estimated the transverse ρ-contribution, (that is the A_{1T} term for ρ-production).

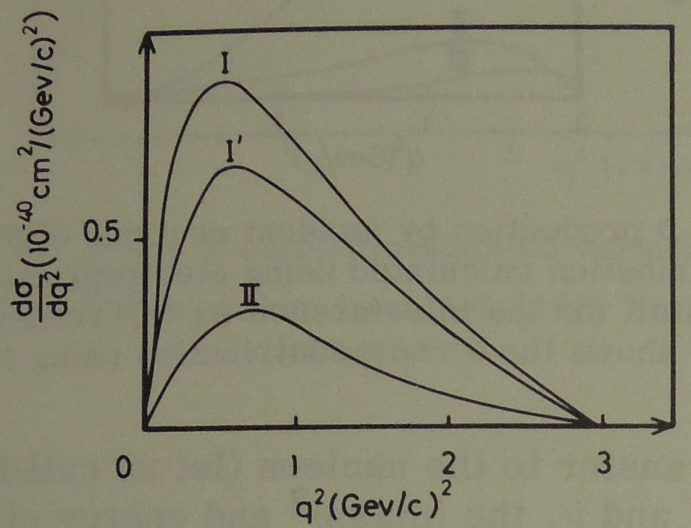

Fig. 8. q^2 distribution for A_1 production by incident neutrino of 3 GeV. Curves I (and I') show the diffractive-like A_1 contribution with $\sigma_L \sim \sigma_T$ (and using (26) for σ_L). Curve II shows an upper limit for the interference $\rho \to A_1$ transverse contribution.

We see that for a fixed incident neutrino energy E the production of A_1 by neutrinos is less than that for ρ, keeping equal the coupling constants $g_{\ell\rho}$ and $g_{\ell A_1}$ and elastic ρP and A_1P cross sections. It comes from the fact that the coupling constants enter into the amplitude as $g/(m^2 + q^2)$ where m is the mass of the meson and g its weak-coupling decay and for foreseeable experimental energies, moderate $q^2 (q^2 \sim 1(\mathrm{GeV}/c)^2)$ will be dominant so that the effect of heavier A_1 mass in the propagator will make cross sections for the axial current smaller than that of the vector. Below, with these assumptions, we find, for example, that neutrino ρ-production is about 1.5 times A_1 production (see fig. 10, curves I and II). Note that this should hold as well for the total cross section (i.e. into all channel) coming from the axial current as compared with that coming from the vector current.

(iii) Weak π-production. For π-production, we have again (21) for small q^2. At large q^2 where the propagators have dropped off, the constant-with-energy diffraction-like contribution to π-distribution must come from longitudinal A_1. Taking experimental numbers from the physical region, since this is an inelastic process, its cross section is not very large (0.2 mb) and we have a very small π-production for q^2 larger than few m_π^2 (fig. 9, curve I). A much larger result comes from applying (26) away from $q^2 \sim 0$ (curve I'). This is the one process where our presumably dominant contribution comes from the behaviour of the little known longitudinal amplitudes.

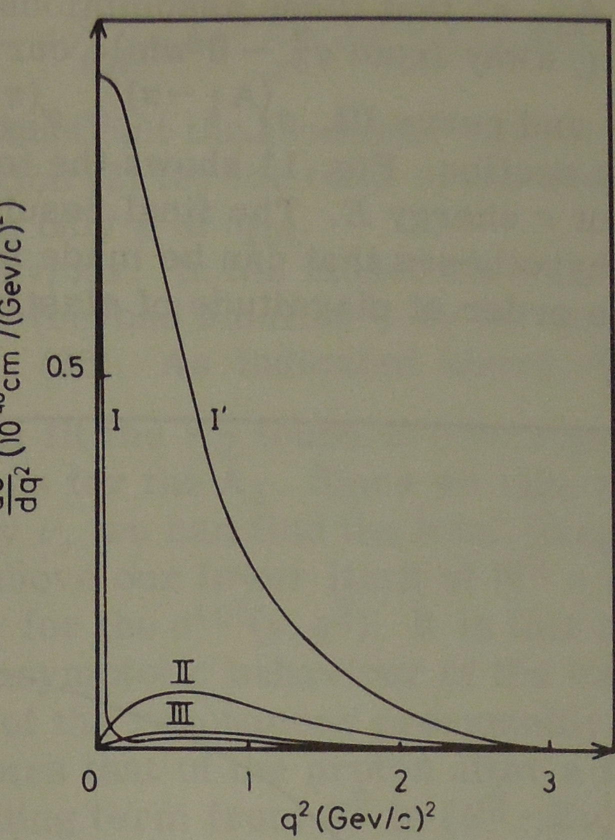

Fig. 9. q^2 distribution for π production by incident neutrino of 3 GeV. Curves I (and I') show the diffractive-like π- and A_{1L} contributions with $\sigma_L = \sigma_{exp}^{(\pi \to A_{1L})}$ (and using (26) for σ_L). Curves II (and III) show the longitudinal (and transverse) ρ-contributions.

It will be interesting to see if in fact single π-events are relatively rare in high ω^2 region away from very small q^2. The vector contribution can be estimated from experimental data in $\pi P \to \rho P$ (mainly longitudinal, curve II), while ρ^0 photoproduction indicates the transverse ρ-contribution (curve III).

(iv) Total weak meson production. Fig. 10 shows the kind of integrated

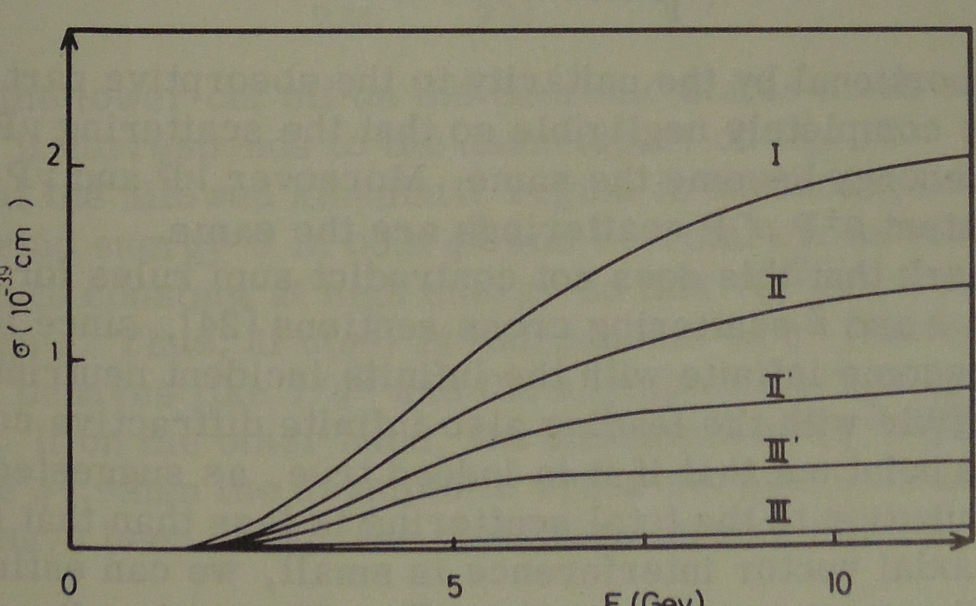

Fig. 10. Cross sections for $\rho^{\pm}$, $A^{\pm}$, $\pi^{\pm}$ production by neutrino as a function of neutrino energy. Curve I shows the ρ-production. Curves II (and II') show the A_1 production with $\sigma_L \sim \sigma_T$ (and σ_L given by (26)). Curves III (and III') show the π-production with $\sigma_L^{(A_1 \to \pi)} = \sigma_{exp}^{(\pi \to A_1)}$ (and σ_L given by (26)).

cross sections for $\rho^{\pm}$, $A_1^{\pm}$, $\pi^{\pm}$ that these assumptions lead to. Curves II' and III' using (26) for σ_L away from $q^2 \sim 0$ while curve II uses $\sigma_L^{(A_1 \to A_1)} \simeq \sigma_T^{(A_1 \to A_1)}$ and curve III, $\sigma_L^{(A_1 \to \pi)} \simeq \sigma_{exp}^{(\pi \to A_{1L})}$.

(v) Total weak cross section. Fig. 11 shows the total ν cross section as a function of the incident ν energy E. The final result turns out to be independent of the various hypotheses that can be made for the longitudinal cross section and of the order of magnitude of elastic and quasi-elastic cross sections [23].

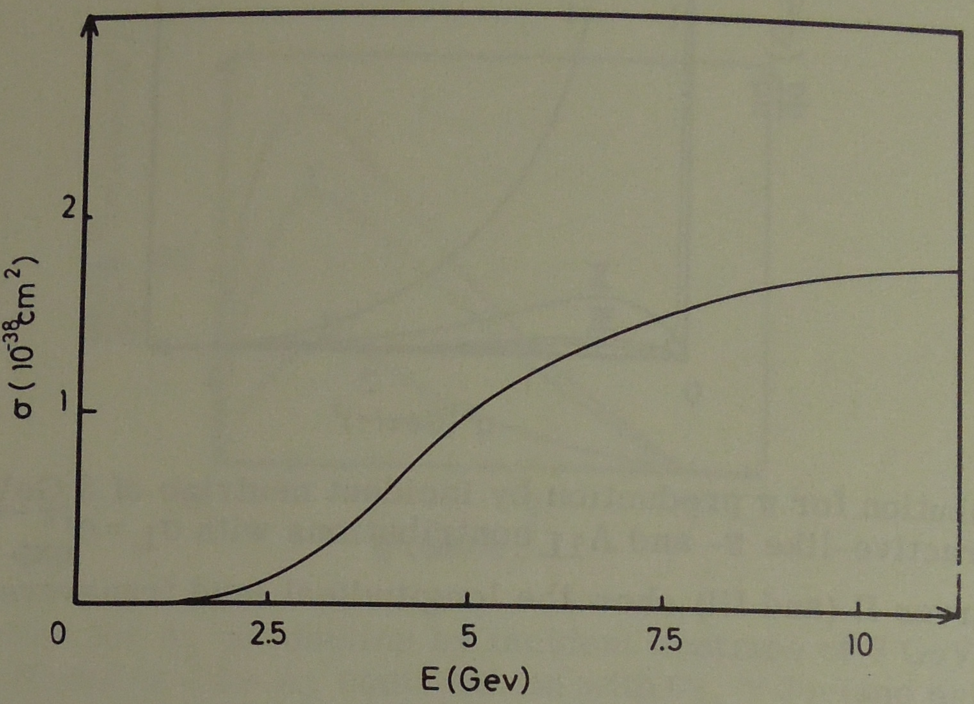

Fig. 11. Total ν cross section as a function of neutrino energy. Taking for σ_L the experimental numbers from the physical region or applying (26) away from $q^2 = 0$ gives the same result.

The axial-vector interference terms

$$\sum_F V_\mu^F \cdot \epsilon_\mu^T \; A_\nu^{*F} \cdot \epsilon_\nu^{*T}$$

which are proportional by the unitarity to the absorptive part of $\rho^T \to A_1^T$ turns out to be completely negligible so that the scattering νP and $\bar{\nu} N$ for high hadronic energy become the same. Moreover νP and $\bar{\nu} P$ are also the same to the extent $\rho^+ P$ $\rho^- P$ scatterings are the same.

Let us remark that this does not contradict sum rules for the differences between ν and $\bar{\nu}$ scattering cross sections [24], since in fact the difference may become infinite with the infinite incident neutrino energy, but remains negligible with the leading also infinite diffractive contribution. Finally, let us point out that if it is indeed true, as suggested above, that the axial contribution to the total scattering is less than that of the vector, and since the axial vector interference is small, we can estimate crudely the total neutrino cross section in the diffractive region directly from electroproduction total cross section, by adding perhaps 50% to 75% to account for the axial current and write a formula similar to (43) for the ρ diffractive production:

$$d\sigma_{\text{tot}}^{(\nu)} = 4q^4 \frac{G^2}{e^4} d\sigma_{\text{tot}}^{(e)} (1+Ax) . \tag{44}$$

In principle we should subtract the isoscalar (ω) part from $d\sigma^{(e)}$, but the vector dominance relation [2] for the total cross section in fact indicates that this contribution is only $\frac{1}{9}$ of σ_{tot}.

(vi) Asymptotic behaviour. In the model the total cross section at a given q^2 and ν is given (beyond small q^2) by ρP and A_1P total cross sections inserted into eq. (36). As indicated above $\sigma_T^{\text{tot}}(\rho P) = \sigma_L^{\text{tot}}(\rho P) \simeq 31$ mb, will fit the W_2 found in electroproduction and we shall assume the same figures for the A_1. Since we take these cross sections to be constant with energy ν, we can find the total integrated ν cross section coming from masses above our lower limit of $W^2 = 3\,(\text{GeV})^2$, by assuming a definite q^2 behaviour for the $\sigma^{\text{tot}}(\nu, q^2)$. It is this contribution, of course, which determines the asymptotic behaviour of the total cross sections, since the contribution of the resonances presumably levels off at the contribution of several times that of the proton after a few GeV. As can be seen from (36) the leading term from E^2 in $(E^2 - E\nu)$ and the essential integral to be evaluated is then of the form:

$$\int dq^2\, d\nu \frac{q^2}{(q^2 + m_\rho^2)^2} \frac{\sigma(q^2, \nu)}{\nu} ,$$

(since $|\boldsymbol{q}| \simeq \nu$ for large ν).

The allowed kinematics for q^2 and ν is:

$$0 \leqslant q^2 \leqslant 4E(E - \nu) ,$$

$$\frac{q^2}{2M} + \frac{\omega^2 - M^2}{2M} \leqslant \nu ,$$

where ω^2 is the lower cut off on the hadronic states mass squared in our calculations. It corresponds to the dashed part on fig. 12.

We see that the allowed kinematic region is expanding linearly with the incident neutrino energy E in both q^2 and ν. Total cross sections are expected to tend to constant at high energy so that $\sigma(q^2, \nu)$ must be independent of ν at high ν. Thus, if $\sigma(q^2)$ is independent on q^2 (naive ρ-dominance), the integrand behaves like $1/q^2$ and the asymptotic cross section is: $\sim (\log E/M)^2$. If on the other hand, the assumption $\sigma(q^2)/q^2 \sim$ constant holds at large q^2, then the asymptotic cross section $\sim E$. If we put in the numbers we have been using, the coefficient comes out:

$$\left.\begin{array}{l} \sigma \sim 0.5 \cdot 10^{-38}\ \text{cm}^2\ (\log E/M)^2\ (1+Ax) \\ \sigma \sim 0.8 \cdot 10^{-38}\ \text{cm}^2\ E/M\ (1+Ax) \end{array}\right\} E \to \infty . \tag{45}$$

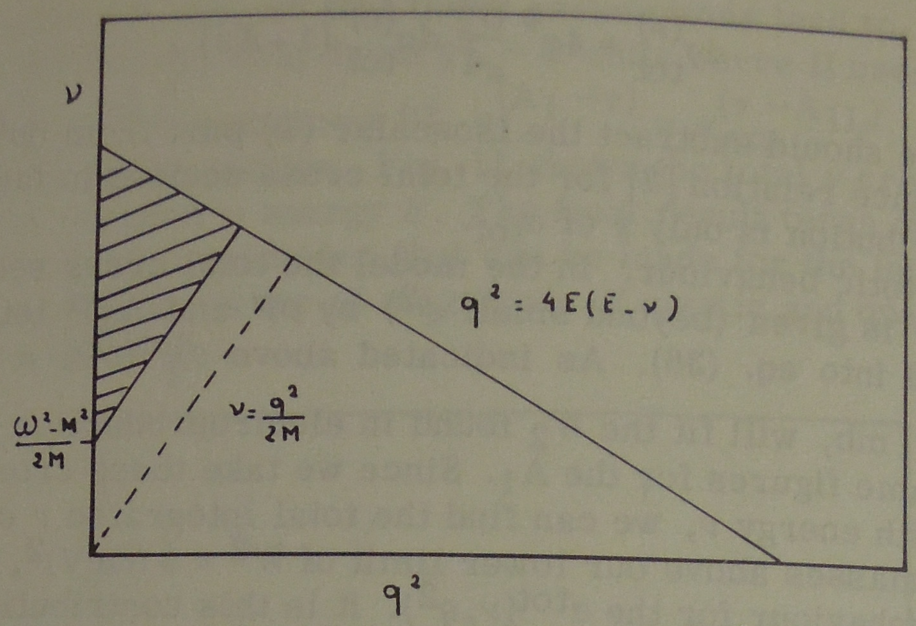

Fig. 12. Allowed kinematic region for q^2 and ν.

The relative axial contribution, less than that of the vector in the region of interest, has been indicated explicitly. Eventually, at least in the model where $q^2/(q^2+m_\rho^2)^2$ tend to constant/q^2, the cross section eventually becomes insensitive to the mass in the propagator, but this requires very high E since the allowed kinematic region favours small q^2. Compare fig. 10 where the ρ and A_1 production curves I and II are directly proportional (by a factor $\frac{1}{6}$) to the vector and axial contributions to the total cross sections.

5. CONCLUSION

To conclude, let us first note some of the general characteristics of the model.

(i) Production of high-energy hadronic systems should have the general character of high-energy meson-nucleon collisions (for various types of particles and momentum transfer distributions) with the leptonic momentum-transfer direction as the beam direction.

(ii) The largest single modes of two-body production should be the final states $\{F\} = (\rho^\pm P,\ A_1^\pm P,\ \pi^\pm P)$, the ± is for ν or $\bar{\nu}$ incident (or with neutron for nuclear targets). We already know from photoproduction experiments that the ρ plays a large role in the vector current; by CVC we should necessarily expect it in the weak vector current also. The occurrence of significant A_1 production would support the idea that it is the chiral partner of the ρ. Leptonic single π-production for high hadronic mass should be very rare for q^2 larger than few m_π^2 (due to the rapid decrease of the π-propagator).

(iii) We have seen that the main uncertainty comes from the ambiguity of the unphysical mass squared behaviour of the longitudinal cross sections away from zero mass, so that in both electro- and neutrino-production experiment, it would be interesting to study the polarization of the produced ρ to see what role is in fact played by longitudinal polarization.

(iv) We expect vector-axial interference to become small since the cross section for this, an interference term like $\Sigma_F\, T^{(\rho\to F)} T^{*(A_1\to F)}$ is proportional by unitarity condition to the absorptive part of $\rho\mathcal{N}\to A_1\mathcal{N}$, which being a non-diffractive process should become small at high energy. Thus the difference between νP and $\bar{\nu}N$ scattering should become small. νP and $\bar{\nu}P$ are also the same to the extent that ρ^+P, ρ^-P scattering are the same. Again we speak of the region where the energy of $\{F\}$ is high.

(v) There is an important general feature which should be a severe test of any model of this type: for a given large mass ω of the hadronic system, the qualitative aspects of the system should not change rapidly with q^2, (aside from explicitly indicated π-propagator contribution in ν-reactions), since we make the assumption that essentially the same states are excited, independent of q^2, although with varying strength perhaps, the nature of the state at a given large ω should be roughly q^2 independent. For example, if multiplicities or say strangeness production were found to vary rapidly with q^2 at large ω in electroproduction, then another model than the simple type used here would be indicated.

(vi) For processes where the effect of the axial current is small according to the model, as in ρ-production, using CVC and assuming that the amplitudes involved in the action of the isovector, vector current are isospin independent at high energy, then we can have a certain degree of model independence by relating neutrino production directly to electroproduction by way of:

$$\frac{d\sigma^{(\nu\to\rho^+)}}{dq^2\, d\nu} \simeq 4q^4\, \frac{G^2}{e^4}\, \frac{d\sigma^{(e-\rho^0)}}{dq^2\, d\nu}\,.$$

In the same way since the axial contribution to the total scattering seems to be less than that of the vector and since the vector-axial interference is small, this formula can also be crudely used to estimate the total neutrino cross section by adding perhaps 50 to 75% to account for the axial current.

$$d\sigma^{(\nu)}_{tot} \sim 4q^4\, \frac{G^2}{e^4}\, \sigma^{(e)}_{tot}\, (1+Ax)\,.$$

Finally let us recall that, independently of the model, we have derived simply an Adler type formula for weak scattering at small q^2 where the cut-off on scattering angle is not required so that the ambiguities of choosing the forward direction are suppressed.

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NOTE ADDED IN PROOF

After completing this work we received new experimental data from SLAC-MIT on deep electroproduction for high q^2 values [25]; more precisely at 6° and 10° scattering angles for invariant hadronic masses $\omega = 2$, 3, 3.5 GeV, the variations with q^2 of $d^2\sigma/d\Omega dE'/(d\sigma/d\Omega)_{\mathrm{Mott}}$ i.e. $W_2 + 2W_1 \mathrm{tg}^2 \frac{1}{2}\theta$ are studied. For low q^2 values up to $q^2 \sim 4(\mathrm{GeV}/c)^2$ all the experimental data are in good quantitative agreement with the diffraction viewpoint of the ρ-dominance. The discrepancy observed for the highest q^2 value (namely a factor two for $q^2 = 7(\mathrm{GeV}/c)^2$, $\omega = 2$ GeV) does not invalidate the model since the condition $\nu \gg \sqrt{q^2}$ is no longer true.

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